PHY1112: Assignment 8

> Matrix Madness: Episode 2

Assigned: March 5th, 2024

Due: March 12th, 2024

Learning Objectives

1. Practice linear algebra in Python
2. Learn about the two-dimensional rotation matrix

Grade Breakdown

|  |  |  |
| --- | --- | --- |
| Part | 1 | Total |
| Points | 23 | 23 |
| Score |  |  |

**Question 1: Rotation Repetition.**

Consider the following matrix,

that rotates a two-dimensional vector in a Cartesian basis, by some angle . is called a rotation matrix.

If the two dimensions of the vector are , then the rotation will occur around the axis, like in the following diagram (where the direction is out of the page),

axis

axis

1. Write a function in Python that returns the rotation matrix as a NumPy array of shape (2,2) for an arbitrary angle , where is to be provided as input to your function.

Use your function to calculate for radians. Print your result to the terminal and include a snapshot in your solutions document.  
**(3 marks)**

1. Rotate the vector

by an angle of radians to obtain a new vector Perform the matrix-vector multiplication using `np.dot`. Print the answer to the terminal and provide a snapshot of your results.   
**(2 marks)**

1. Using `np.linalg.inv`, calculate , which is the inverse of . Then, using `np.transpose`, calculate , which is the transpose of . Print out both results to the terminal and include a snapshot of your results. What do you notice?

**(3 marks)**

1. Now, take the matrix product of and its inverse . Then, take the matrix product of and Print out both results to the screen and include a snapshot of your results. What do you notice?  
   **(3 marks)**
2. Based on your result in part ‘d’, what result would you expect when you apply to your rotated vector from part ‘b’, Verify your hypothesis by using your script to calculate .   
   **(2 marks)**
3. The formula for the angle between two vectors and is given by:

Write a function that takes in two vectors as input and returns the angle , using NumPy to implement the formula. Use your function to determine by which angle calculated in part ‘b’ needs to be rotated to make it orthogonal to

If necessary, add or subtract factors of to your result to report a value in the range .

**Hint**: It is easier to use this formula to determine what angles these vectors make with the x-axis, rather than with each other.

Test that your determined angle does indeed make the rotated orthogonal to , by applying the rotation matrix to and taking the inner product of and . Print the result of the inner product to the screen, such that only 8 decimal places are displayed.  
**(5 marks)**

1. Using `np.linalg.eig`, determine the eigenvalues and eigenvectors for What do you notice about the eigenvectors?   
   **(4 marks)**

**(23 marks total, 1 for docstrings/file header/variable naming/comments)**